COMPLETE NULL DATA FOR A BLACK HOLE COLLISION

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We discuss a sequence of numerically constructed geometries describing binary black hole event horizons – providing the necessary input for characteristic evolution of the exterior spacetime. Our sequence approaches a single Schwarzschild horizon as one limiting case and also includes cases where the horizon's crossover surface is not hidden by a marginally anti-trapped surface (MATS).

In previous work, we presented a conformal horizon model of a binary black hole which generates the "pair-of-pants" event horizon structure found in the axisymmetric head-on collision of black holes [1]. The conformal model constructs a null surface geometry by conformal rescaling of the Minkowski lightcone of a spheroid of eccentricity ϵ, and supplies part of the data for a characteristic evolution *backward in time* along ingoing null hypersurfaces. The strategy and details behind this new approach to determine a part of the exterior space-time and the waveform emitted in the post-merger phase has been outlined elsewhere [1,2,3,4]. Recently we generalized our previous study of the intrinsic geometry of the horizon to a full stand-alone description of both the intrinsic and extrinsic horizon data necessary to implement a characteristic evolution [4]. In such an evolution of the exterior spacetime, along a family of *outgoing* null hypersurfaces, we choose the inner boundary as the null worldtube representing a *white hole horizon*. This horizon pinches off in the future, where its generators either caustic or cross each other (such as they do at the vertex of a null cone). We work in the coordinate system first introduced by Sachs to formulate the double-null characteristic initial value problem [6].

In the close approximation [5] regime of a white hole fission, when reinterpreted in the time reversed sense of a black hole merger, we find that the individual black holes merge inside a white hole horizon corresponding to the marginally anti-trapped branch of the $r = 2M$ Schwarzschild surface. In [4] we demonstrate numerically that in the non-perturbative regime an entirely different scenario is possible, in which the individual black holes form and merge without the existence of a MATS on the event horizon. Since the Bondi surface area coordinate is singular on a MATS, the absence of a MATS is required for a Bondi evolution backward in time throughout the space-time region exterior to the black holes. In our present approach, we deal with the Bondi boundary $\mathcal{B}$, defined by $\partial_\lambda r = 0$, rather than the MATS. We prove in [4] that a marginally trapped surface cannot form before the Bondi boundary on a white hole horizon, thus in the time reversed black hole picture, absence of a Bondi boundary implies absence of a MATS.

Of special physical importance is the location of the *crossover surface* $\mathcal{X}$, where the horizon pinches off, relative to the surface $\mathcal{B}$. For small ϵ the fission (located on the equator of $\mathcal{X}$) is "hidden" beyond $\mathcal{B}$ in the sense that it is not visible to observers at $\mathcal{I}^+$. From the view of a black hole merger, the individual black holes would merge inside a white hole horizon. Our analytical results described in [4] suggest that sufficient nonlinearity might cause the white hole fission to occur prior to $\mathcal{B}$. This scenario can indeed be demonstrated by numerical integration of the equations underlying the conformal horizon model. At an early time, the equilibrium conditions on the white hole horizon imply that $r = 2M$ and $\partial_\lambda r = -u/4M > 0$ (u is a suitably normalized affine coordinate at the horizon). As the horizon evolves, the surface area r decreases along all rays. The outward expansion measured by $\Theta_{OUT} = 2\partial_\lambda r/r$ also initially decreases along all rays, although this process can be reversed by the growth of nonlinear terms, as indicated by the ray-averaged behavior. In the close approximation, the expansion goes to zero along all rays before the horizon pinches off, i.e. the crotch at the center of the pair of pants is hidden behind a MTS. The crucial question in the nonlinear regime is whether the horizon can pinch off before the formation of a Bondi boundary, i.e. the issue is who wins the race towards zero, the radius r or the expansion Θ_{OUT} along some ray.

We have conducted this race for a sequence of models in the range $0 \leq \epsilon \leq 10^{-2}$ and monitored the minimum value of the expansion of the outgoing null rays on the horizon over the sphere, and of the Bondi radius of the horizon. We find that indeed for small ϵ we can confirm the close limit picture where a Bondi boundary forms before the radius has changed significantly. But for sufficiently large eccentricity ($\epsilon = 10^{-3}$), the radius *does* win the race by a sudden plunge to zero before the expansion has undergone any appreciable change. This is a genuine nonlinear effect: While the expansion begins with a flying start (its initial slope) and gets accelerated by linear effects, the radius starts from rest and "only" gets accelerated by quadratic or higher nonlinearities.

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References

1. L. Lehner *et al.*, Phys. Rev. D **60**, 044005 (1999).
2. S. Husa and J. Winicour, Phys. Rev. D **60**, 084019 (1999).
3. J. Winicour, Prog. of Theor. Phys. Supp. **136**, 57 (1999).
4. Roberto Gomez, Sascha Husa and Jeffrey Winicour, gr-qc/0009092.
5. R. Price and J. Pullin, Phys. Rev. Lett. **72**, 3297 (1994).
6. R. Sachs, J. Math. Phys. **3**, 908 (1962).